Azimuthal asymmetries at HERA: theoretical aspects

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Abstract

We comment on theoretical aspects of the measurement of azimuthal asymmetries in semi-inclusive charged particle production, made recently by the ZEUS Collaboration at HERA. By taking the ratio of the two measured asymmetries, we find good agreement between the perturbative QCD prediction and the experimental data. To separate the perturbative and nonperturbative contributions to the asymmetries, we suggest that the azimuthal asymmetries of the transverse energy flow be measured as a function of a variable q_T related to the pseudorapidity of the energy flow.

Key words: Semi-inclusive deep inelastic scattering; QCD; resummation
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In a recent publication [1] the ZEUS Collaboration at DESY-HERA has presented data on asymmetries of charged particle ($h^{\pm}$) production in the process $e + p \xrightarrow{\gamma^*} e + h^{\pm} + X$, with respect to the angle φ defined as the angle between the lepton scattering plane and the hadron production plane (of $h^{\pm}$ and the exchanged virtual photon). The azimuthal asymmetries, $\langle \cos \varphi \rangle$ and $\langle \cos 2\varphi \rangle$, as functions of the minimal transverse momentum p_c of the observed charged hadron $h^{\pm}$ in the hadron-photon center-of-mass (hCM) frame, are defined as

$$\langle \cos n\varphi \rangle (p_c) = \frac{\int d\Phi \int_0^{2\pi} d\varphi \cos n\varphi \frac{d\sigma}{dxdzdQ^2 dp_T d\varphi}}{\int d\Phi \int_0^{2\pi} d\varphi \frac{d\sigma}{dxdzdQ^2 dp_T d\varphi}}, \qquad (1)$$

with $n = 1, 2$. In terms of the momenta of the initial proton P^{μ}, the final-state hadron P_h^{μ}, and the exchanged photon q^{μ}, the variables in (1) are $Q^2 = -q_{\mu} q^{\mu}$, $x = Q^2/2(P \cdot q)$, and $z = (P \cdot P_h)/(P \cdot q)$. $\int d\Phi$ denotes the integral over x, z, Q^2, p_T within the region defined by $0.01 < x < 0.1$, $180 \text{ GeV}^2 < Q^2 < 7220 \text{ GeV}^2$, $0.2 < z < 1$, and $p_T > p_c$. Nonzero $\langle \cos 2\varphi \rangle$ comes from interference of the helicity $+1$ and -1 amplitudes of the transverse photon polarization; and nonzero $\langle \cos \varphi \rangle$ comes from interference of transverse and longitudinal photon polarization.

More than 20 years ago it was proposed to test QCD by comparing measured azimuthal asymmetries to the perturbative predictions [2]. However, it was also realized that nonperturbative contributions and higher-twist effects may affect the comparison [3–6]. For example, intrinsic k_T might be used to parametrize the nonperturbative effects [3], and indeed ZEUS did apply this idea to their analysis of the data [1]. The relative importance of the nonperturbative effects is expected to decrease as p_T increases. Thus, the azimuthal asymmetries in semi-inclusive deep-inelastic scattering (SIDIS) events with large p_T should be accurately described by perturbative dynamics. From the comparison to the perturbative QCD calculation at the leading order in α_s [7,8], the ZEUS Collaboration concluded that the data on the azimuthal asymmetries at large values of p_c, although not well described by the QCD predictions, do provide clear evidence for a perturbative QCD contribution to the azimuthal asymmetries.

In this paper, we will take a new look at the ZEUS data, motivated by a QCD resummation formalism [9–12] that takes into account the effects of multiple soft parton emission. First, we argue that the analysis of $\langle \cos\varphi \rangle$ and $\langle \cos 2\varphi \rangle$ based on fixed-order QCD is unsatisfactory because it ignores large logarithmic corrections due to soft parton emission. We also show that perturbative and nonperturbative contributions are mixed in the transverse momentum distributions. Then we make two suggestions for improvement of the analysis of the ZEUS data. We show that perturbative and nonperturbative contributions can be separated more clearly in asymmetries depending on a variable q_T related to the pseudorapidity of the final hadron in the hCM frame. We also suggest measurement of the asymmetries of the *transverse energy flow* which are simpler and may be calculated reliably. Our predictions for the asymmetries of transverse energy flow are the most important contribution of this paper.

1 Large logarithmic corrections and resummation

The impact parameter resummation formalism that we are applying here describes production of nearly massless hadrons in the current fragmentation region, where the production rate is the highest. In this region, transverse momentum distributions are affected by large logarithmic QCD corrections due to radiation of soft and collinear partons. The leading logarithmic contributions can be summed through all orders of perturbative QCD [10–12] by applying a method originally proposed in [9] for jet production in $e^+ e^-$ annihilation and the Drell-Yan process.

The spin-averaged cross section for SIDIS in a parity-conserving channel, *e.g.*, γ^* exchange, can be decomposed into a sum of independent contributions from four basis functions $A_\rho(\psi, \varphi)$ of the leptonic angular parameters ψ, φ [13]:

$$\frac{d\sigma}{dx\,dz\,dQ^2\,dq_T^2\,d\varphi} = \sum_{\rho=1}^{4} {}^\rho V(x, z, Q^2, q_T^2) A_\rho(\psi, \varphi).$$

Here ψ is the angle of a hyperbolic rotation (a boost) in Minkowski space; it is related to the conventional DIS variable y, by $y = P \cdot q / P \cdot \ell = 2/(1 + \cosh \psi)$. The angular basis functions are $A_1 = 1 + \cosh^2 \psi$, $A_2 = -2$, $A_3 = -\cos \varphi \sinh 2\psi$, $A_4 = \cos 2\varphi \sinh^2 \psi$. Of the four structure functions ${}^p V$, only ${}^1 V$ and ${}^2 V$ contribute to the denominator of (1), *i.e.*, the φ-integrated cross section. Of these two terms, ${}^1 V$ is more singular and it dominates the rate. To explore the singular contributions in ${}^1 V$, we introduce a scale q_T related to the *polar* angle (θ) of the direction of the final hadron in the hCM frame. A convenient definition is

$$ q_T = Q \sqrt{1/x - 1} \, \exp{(-\eta)}, \tag{2} $$

where η is the pseudorapidity of the charged hadron in the hCM frame (defined with respect to the direction of the momentum q^μ of γ^*). In the limit $q_T \to 0$, the structure function ${}^1 V$ is dominated by large logarithmic terms; it has the form $q_T^{-2} \sum_{k=1}^{\infty} (\alpha_s/\pi)^k \sum_{m=0}^{2k-1} v^{(km)} \ln^m(q_T^2/Q^2)$, where $v^{(km)}$ are some generalized functions. To obtain a stable theoretical prediction, these large terms must be resummed through all orders of perturbative QCD. The other structure functions ${}^{2,3,4} V$ are finite at this order; we approximate them by fixed-order $O(\alpha_S)$ expressions.

In Eq. (1), the numerator of $\langle \cos \varphi \rangle$ or $\langle \cos 2\varphi \rangle$ depends only on the structure function ${}^3 V$ or ${}^4 V$, respectively. The measurement of $\langle \cos \varphi \rangle$ or $\langle \cos 2\varphi \rangle$ must be combined with good knowledge of the φ-integrated cross section, *i.e.*, the denominator of (1), to provide experimental information on the structure function ${}^3 V$ or ${}^4 V$. Thus it is crucial to check whether the theory can reproduce the φ-integrated cross section as a function of p_T before comparing the prediction for (1) to the data. But, on the contrary, as shown in [12], the $\mathcal{O}(\alpha_s)$ fixed-order cross section is significantly lower than the data from [14] in the range of p_T relevant to the ZEUS measurements. This difference signals the importance of higher-order corrections and undermines the validity of the $\mathcal{O}(\alpha_s)$ result as a reliable approximation for the numerator of (1).

On the other hand, the resummation calculation [12] with a proper choice

of the nonperturbative function yields a much better agreement with the experimental data for the φ-integrated p_T-distribution from [14]. One might try to improve the theoretical description of the ZEUS data using resummation for the denominator of (1). However, the resummation calculation for $d\sigma/(dxdzdQ^2dp_Td\varphi)$ in the phase space region relevant to the ZEUS data is currently not possible, largely because of the uncertainty in the parameterization of the nonperturbative contributions in this region. The resummed structure function ^1V includes a nonperturbative Sudakov factor, which contains the effects of the intrinsic transverse momentum of the initial-state parton and the nonperturbative fragmentation contributions to the transverse momentum of the final-state hadron. Without first determining this nonperturbative factor, $e.g.,$ from other measurements, it is not possible to make a trustworthy theoretical prediction for the denominator of (1) and, hence, these azimuthal asymmetries.

The azimuthal asymmetries measured by ZEUS may also be sensitive to uncertainties in the fragmentation to $h^\pm$ in the final state. Indeed, the cross section in (1) includes convolutions of hard scattering cross sections with fragmentation functions (FFs), integrated over the range $0.2 < z < 1$. Although the knowledge of FFs is steadily improving [16], there is still some uncertainty about their z-dependence and flavor structure for the range of Q relevant to the ZEUS measurement. Therefore the most reliable tests of the theory would use observables that are not sensitive to the final-state fragmentation. The asymmetries $\langle \cos n\varphi \rangle$ would be insensitive to FFs if the dependence on the partonic variable $\hat{z}$ were similar in the hard parts of the numerator and denominator of (1), so that the dependence on the FFs would approximately cancel. (We denote the parton-level quantities by "$\hat{\ }$".) It is shown in Appendix B of [11] that the partonic structure function $^1\widehat{V}$, which dominates the denominator of (1), contains terms proportional to $1/\hat{z}^2$ that increase rapidly as $\hat{z}$ decreases. However, the most singular terms in the partonic structure functions $^{3,4}\widehat{V}$ are proportional to $1/\hat{z}$. Therefore, the dependence on the FFs does not cancel in the azimuthal asymmetries.

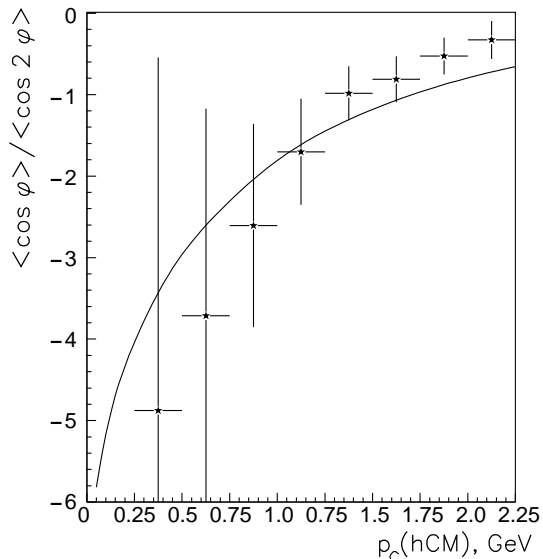A curious fact appears to support the suggestion that the theoretical predic-

Fig. 1. Comparison of the $O(\alpha_s)$ prediction for the ratio $\langle\cos\varphi\rangle/\langle\cos 2\varphi\rangle$ with the ratio of experimentally measured values of $\langle\cos\varphi\rangle$ and $\langle\cos 2\varphi\rangle$ from [1]. The error bars are calculated by adding the statistical errors of $\langle\cos\varphi\rangle$ and $\langle\cos 2\varphi\rangle$ in quadrature. Systematic errors are not included. The theoretical curve is calculated for $\langle x\rangle = 0.022$, $\langle Q^2\rangle = 750$ GeV^2, using the CTEQ5M1 parton distribution functions [15] and fragmentation functions by S. Kretzer from [16].

tions for $\langle\cos n\varphi\rangle$ depend significantly on the fragmentation functions. While each of the measured asymmetries, $\langle\cos\varphi\rangle$ and $\langle\cos 2\varphi\rangle$, deviates from the $\mathcal{O}(\alpha_s)$ prediction, the data actually agree well with the $\mathcal{O}(\alpha_s)$ prediction for the ratio $\langle\cos\varphi\rangle/\langle\cos 2\varphi\rangle$, as shown in Fig. 1. The error bars are the statistical errors on $\langle\cos\varphi\rangle$ and $\langle\cos 2\varphi\rangle$ combined in quadrature; this, however, may overestimate the statistical uncertainty if the two errors are correlated. Since this ratio depends only on the numerators in Eq. (1), which are less singular with respect to $\hat{z}$ than the denominator, the dependence on the fragmentation functions may be nearly canceled in the ratio. The good agreement between the $\mathcal{O}(\alpha_s)$ prediction and the experimental data for this ratio supports our conjecture that the fragmentation dynamics has a significant impact on the individual asymmetries defined in (1).

Our final remark about the azimuthal asymmetries in (1) is that the p_T (or p_c) distributions are not the best observables to separate the perturbative and nonperturbative effects. The region where multiple parton radiation effects are

important is specified by the condition $q_T^2/Q^2 \ll 1$. But the p_T distributions are smeared with respect to the q_T distributions by an additional factor of z, because $p_T = z\, q_T$. Thus the whole observable range of p_T is sensitive to the resummation effects in the region of q_T of the order of several GeV. A better way to compare the data to the perturbative QCD prediction is to express the azimuthal asymmetries as a function of q_T, not p_T. Then the comparison should be made in the region where the multiple parton radiation is unimportant, *i.e.*, for $q_T/Q \gtrsim 1$.

2   Asymmetry of energy flow

Next, we describe an alternative test of perturbative QCD, which will further reduce the above theoretical uncertainties: measurement of the azimuthal asymmetries of the *transverse energy flow*. In the hCM frame, the transverse energy flow can be written as [17,13,10–12]

$$\frac{dE_T}{dx\,dQ^2\,dq_T^2\,d\varphi} = \sum_{\rho=1}^{4} {}^{\rho}V_{E_T}(x, Q^2, q_T^2) A_\rho(\psi, \varphi). \tag{3}$$

Unlike the charged particle multiplicity, the energy flow does not depend on the final-state fragmentation. It has been demonstrated [11,12] that a resummation calculation can provide a good description for the experimental data on the φ-integrated E_T-flow. A new class of azimuthal asymmetries may be defined as

$$\langle E_T \cos n\varphi \rangle(q_T) = \frac{\int d\Phi \int_0^{2\pi} \cos n\varphi \frac{dE_T}{dx\,dQ^2\,dq_T^2\,d\varphi} d\varphi}{\int d\Phi \int_0^{2\pi} \frac{dE_T}{dx\,dQ^2\,dq_T^2\,d\varphi} d\varphi}. \tag{4}$$

The structure functions ${}^{\rho}V_{E_T}$ for the E_T-flow can be derived from the structure functions ${}^{\rho}V$ for the SIDIS cross section [12]. Similar to the case of the particle multiplicities, the asymmetries $\langle E_T \cos \varphi \rangle$ and $\langle E_T \cos 2\varphi \rangle$ receive contributions from ${}^{3}V_{E_T}$ and ${}^{4}V_{E_T}$, respectively. But, unlike the previous case, the denominator in (4) is approximated well by the resummed E_T-flow. Thus these asymmetries can be calculated with greater confidence.
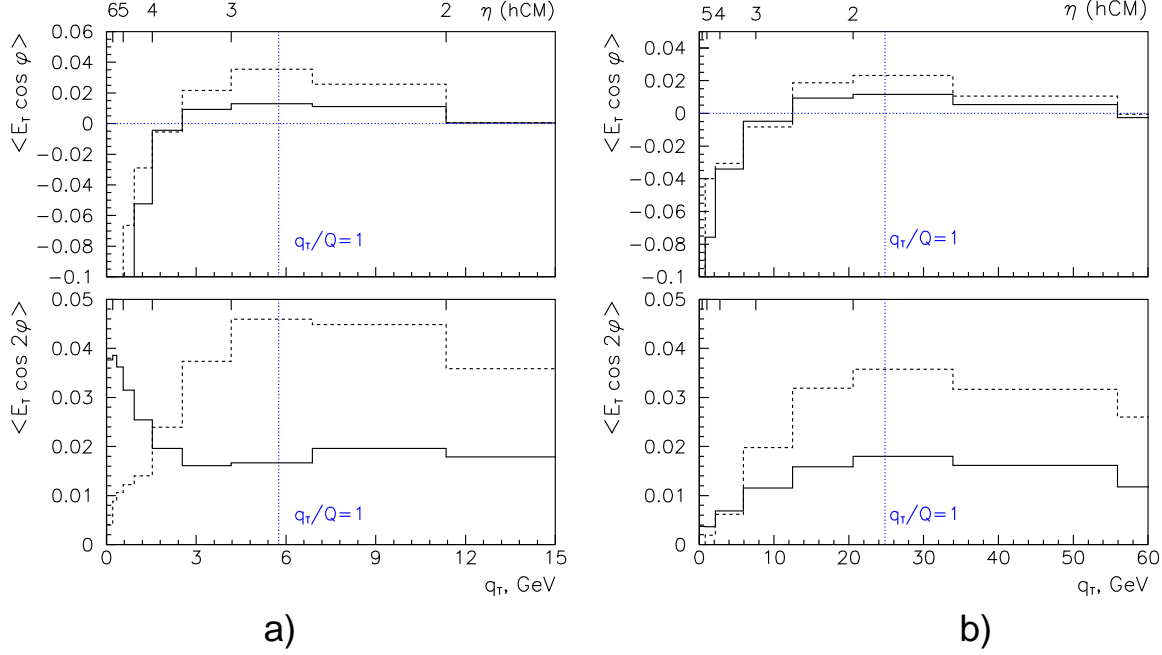
Fig. 2. Energy flow asymmetries $\langle E_T \cos\varphi \rangle(q_T)$ and $\langle E_T \cos 2\varphi \rangle(q_T)$ for (a) $x = 0.0047$, $Q^2 = 33.2$ GeV^2 and (b) $x = 0.026$, $Q^2 = 617$ GeV^2. The Figure shows predictions from the resummed (solid) and the $O(\alpha_s)$ (dashed) calculations.

Figure 2 shows our prediction for the azimuthal asymmetries $\langle E_T \cos\varphi \rangle$ and $\langle E_T \cos 2\varphi \rangle$ as functions of q_T for (a) $x = 0.0047$, $Q^2 = 33.2 \text{GeV}^2$ in the left plots and (b) $x = 0.026$, $Q^2 = 617 \text{GeV}^2$ in the right plots. The asymmetries are shown in q_T-bins that are obtained from the experimental pseudorapidity bins for the φ-integrated E_T-flow data from Ref. [18]. The upper x-axis shows values of the hCM pseudorapidity η that correspond to the values of q_T on the lower x-axis. For each of the distributions in Fig. 2, the structure functions $^3V_{E_T}$ and $^4V_{E_T}$ were calculated at leading order in QCD, i.e., $\mathcal{O}(\alpha_s)$. The solid and dashed curves, which correspond to the resummed and $\mathcal{O}(\alpha_s)$ results respectively, differ because the structure function $^1V_{E_T}$ in the denominator of (4) differs for the two calculations. The resummed φ-integrated E_T-flow is closer to the data than the fixed-order result, so that the predictions made by perturbative QCD for the subleading structure functions $^3V_{E_T}$ and $^4V_{E_T}$ will be confirmed if the experimental azimuthal asymmetries agree with the resummed distributions.

A recent study [12] shows that in the region $q_T \sim Q$ the resummed φ-integrated E_T-flow is larger than the $\mathcal{O}(\alpha_s)$ prediction. This explains why the asymmetries for $q_T \sim Q$ are smaller for the resummed denominator than for the $\mathcal{O}(\alpha_s)$ denominator. In the region $q_T/Q \ll 1$, the asymmetries are determined by the asymptotic behavior of the fixed-order and resummed *partonic* structure functions $^{\rho}\widehat{V}_{E_T}$. As $q_T \to 0$, the $\mathcal{O}(\alpha_s)$ structure functions $(^{1}\widehat{V}_{E_T})_{\mathcal{O}(\alpha_s)}$, $^{3}\widehat{V}_{E_T}$, and $^{4}\widehat{V}_{E_T}$ behave as $1/q_T^2$, $1/q_T$ and 1, respectively. Thus, asymptotically, the ratios $^{3,4}\widehat{V}_{E_T}/(^{1}\widehat{V}_{E_T})_{\mathcal{O}(\alpha_s)}$ go to zero, although the q_T distribution for the asymmetry $\langle E_T \cos\varphi \rangle$ is quite large and negative for small, but non-vanishing q_T (*cf.* Fig. 2). Resummation of $^{1}\widehat{V}_{E_T}$ changes the q_T-dependence of the denominator, which becomes nonsingular in the limit $q_T \to 0$. Consequently, the asymmetry $\langle E_T \cos\varphi \rangle$ with the resummed denominator asymptotically grows as $1/q_T$ (*i.e.*, in accordance with the asymptotic behavior of $^{3}\widehat{V}_{E_T}$). Hence neither the fixed-order nor the resummed calculation for $\langle E_T \cos\varphi \rangle$ is reliable in the low-q_T region, so that higher-order or additional nonperturbative contributions must be important at $q_T \to 0$. The asymptotic limit for the resummed $\langle E_T \cos 2\varphi \rangle$ remains finite, with the magnitude shown in Fig. 2. Since the magnitude of $\langle E_T \cos 2\varphi \rangle$ is predicted not to exceed a few percent, an experimental observation of a large asymmetry $\langle E_T \cos 2\varphi \rangle$ at small q_T would signal the presence of some new hadronic dynamics, *e.g.*, contributions from T-odd structure functions discussed in [6].

Figure 2 shows that the predicted asymmetry $\langle E_T \cos\varphi \rangle(q_T)$ at $q_T/Q = 1$ is about 1–2% for the resummed denominator, while it is about 2–4% for the $\mathcal{O}(\alpha_s)$ denominator. The asymmetry $\langle E_T \cos 2\varphi \rangle(q_T)$ at $q_T/Q = 1$ is about 1.5-2% or 3.5-5%, respectively. Both asymmetries are positive for $q_T \sim Q$. According to Fig. 2a, the size of the experimental q_T bins (converted from the η bins in [18]) for low or intermediate values of Q^2 is small enough to reveal the low-q_T behavior of $^{3,4}V_{E_T}$ with acceptable accuracy. However, for the high-Q^2 events in Fig. 2b, the experimental resolution in q_T may be insufficient for detailed studies in the low-q_T region. Nonetheless, it will still be interesting to compare the experimental data to the predictions of perturbative QCD in the region $q_T/Q \approx 1$, and to learn about the angular asymmetries at large values

of Q^2 and x.

To conclude, we suggest that the azimuthal asymmetry of the energy flow should be measured as a function of the scale q_T. These measurements would test the predictions of the perturbative QCD theory more reliably than the measurements of the asymmetries of the charged particle multiplicity.

Acknowledgments

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